# Year 10
## Shape and Space
## **Arcs and sectors**
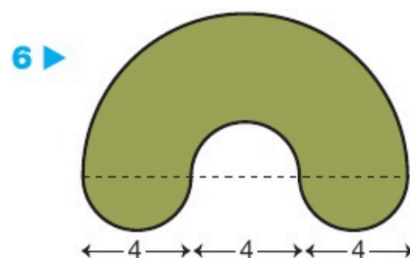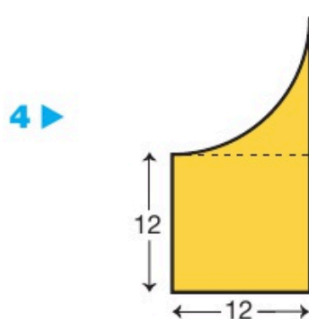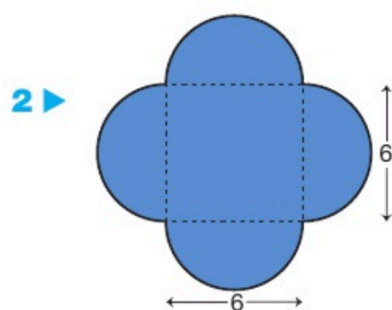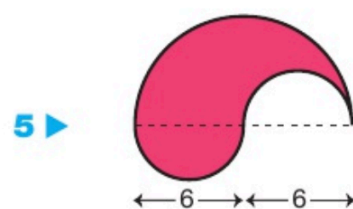
Name _____

House_____

|   |   | Done | S.A |
|---|---|------|-----|
| 1 | Circles | | |
| 2 | Arcs and sectors | | |
| 3 | Problem solving | | |

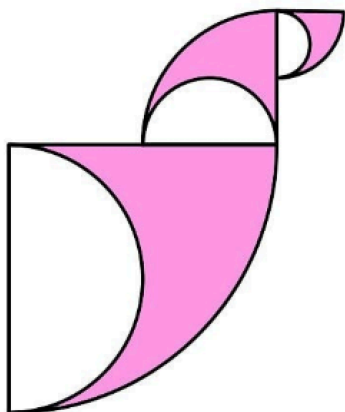Shape and Space III (1) Circles problem solving

**Do now:**

Find the perimeter and area of each of the following shapes, giving answers to 3 s.f. All dimensions are in cm. All arcs are parts of circles.

**1 ▶**

7

**3 ▶**

6

3

**5 ▶**

← 6 → ← 6 →

**2 ▶**

6

← 6 →

**4 ▶**

12

← 12 →
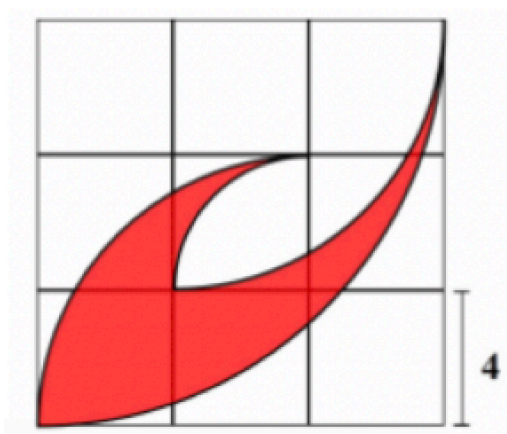
**6 ▶**

← 4 → ← 4 → ← 4 →
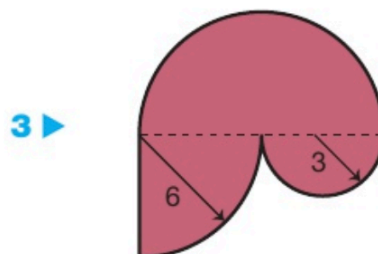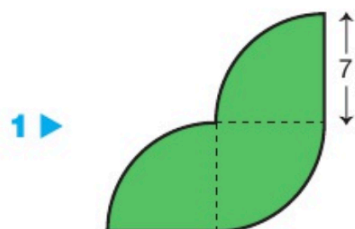
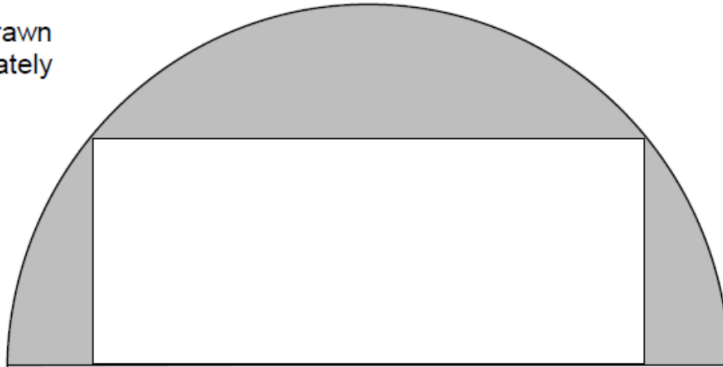Extension

1. What fraction of the shape is shaded?



2. Find the area and perimeter of the shape

3.
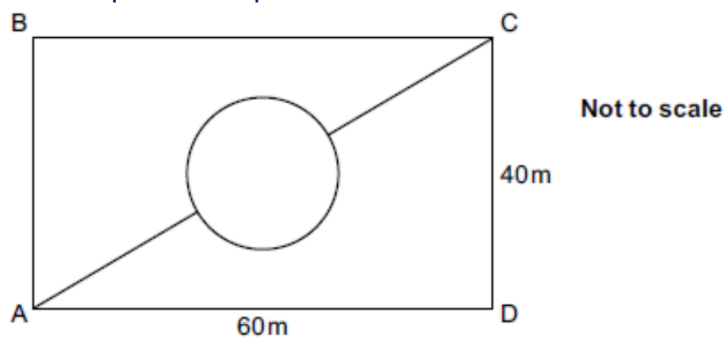. The diagram shows a rectangle inside a semicircle.

Not drawn
accurately



The rectangle has dimensions 16 cm by 6 cm

Work out the shaded area.

Give your answer in terms of $\pi$.

4.
The rectangle ABCD represents a park.
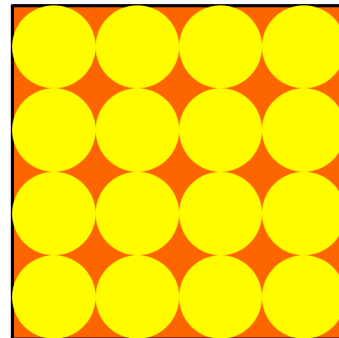


Not to scale

40m

60m

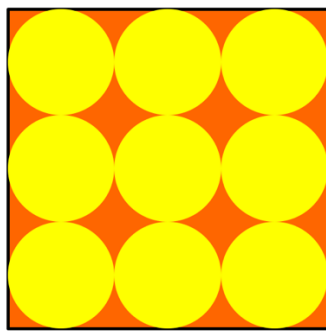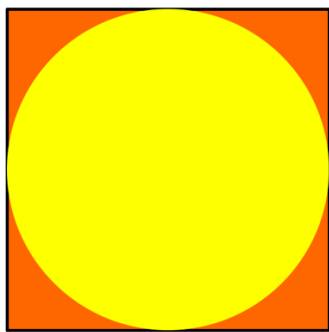The lines show all the paths in the park.

The circular path is in the centre of the rectangle and has a diameter of 10m.

Calculate the shortest distance from A to C across the park, using only the paths shown.

Which has more waste?



Which has the greater shaded area?

(1)
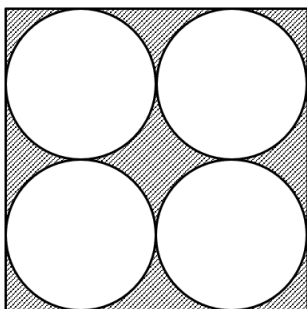


(2)



(3)
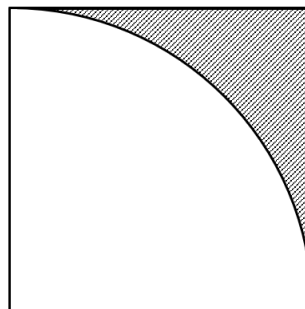


(4)



(5)
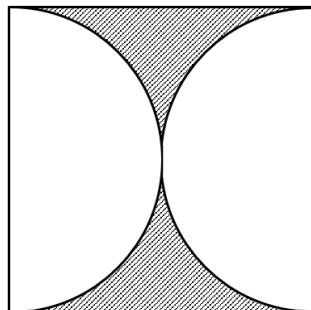


(6)
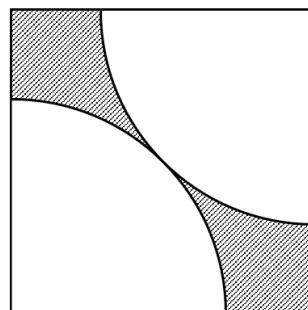


within each of the identical squares, the semicircles and quadrants are the same and they do touch

5

Shape and Space III (2) Sectors and Arcs

**Do now:**

which has the
greater area?
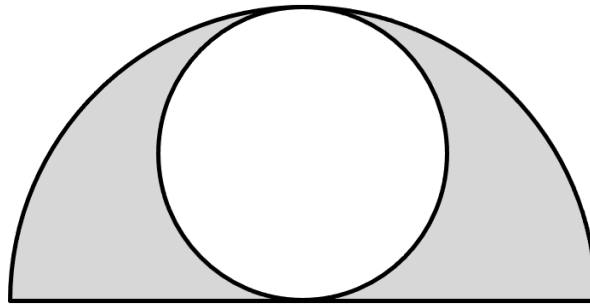
shaded or
unshaded?

How could we calculate the areas and the perimeters of the shapes below?

90°
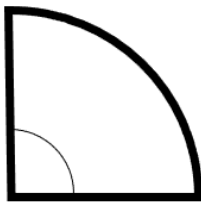
60°
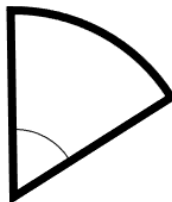
135°

**lengths of arcs (i)**

(1)

A

7cm

90°

B
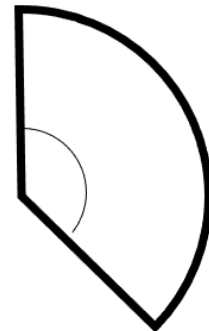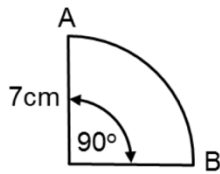
what is the length
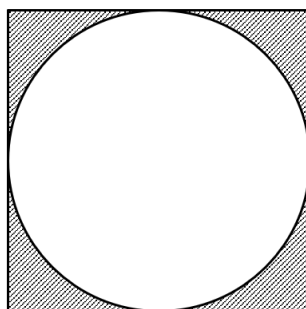of arc AB ?

(2)

C

69.031cm

?°

D
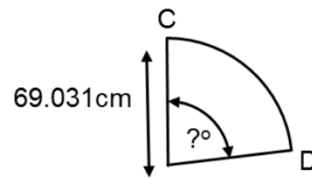
the length of the
arc CD is 100cm

what is the angle?

(3)

E

15cm

?°

F
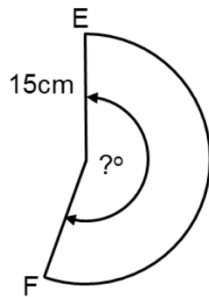
the arc EF is to
have an length of
50cm

what is the angle?

(4) the radius of the London
Eye is 60m

there are 32 equally
spaced capsules

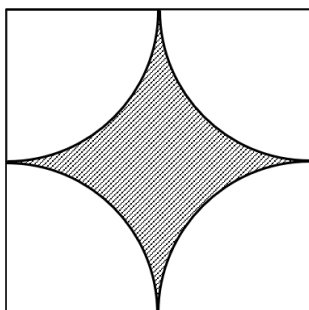how far is it between two
adjacent capsules?

(5)

70m

60m

40°
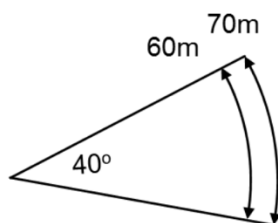
in a 40° (college)
discus throwing area

what is the difference
in arc lengths for the
60m and 70m arcs?

**lengths of arcs (ii)**

(1)
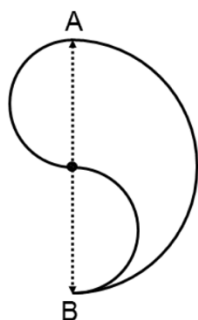
what is the perimeter of the shape if AB is a diameter of the larger circle, of length 20cm?

(2)
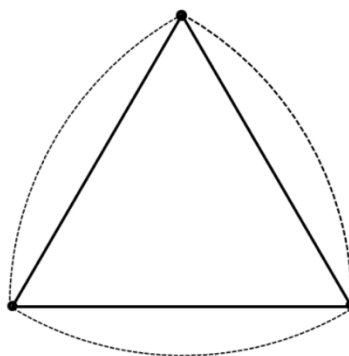
the length of a side of the equilateral triangle is 8cm

what is the total distance around the three arcs (drawn with centres on the corners of the triangle)?

triangle)?

(3)
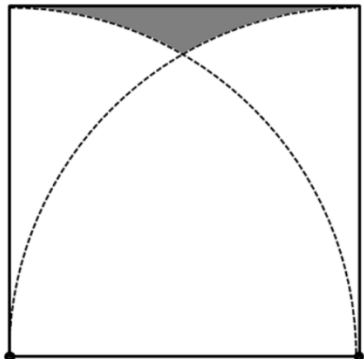
what is the perimeter of the shaded shape if the square has a length of 4cm?

the arcs are drawn with centres on the corners of the square

**areas of sectors (i)**

(1)

6cm  70°  A
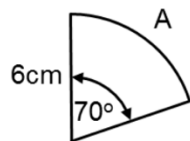
what is the area
of sector A ?

(2)

18cm  197°  B
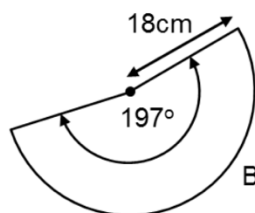
what is the area
of sector B ?

(3)

6cm  ?°  C
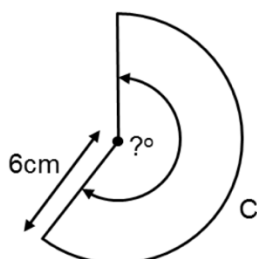
sector C is to have
an area of
60.004cm²

what angle must
sector C have?

(4)

18m  ?°  D
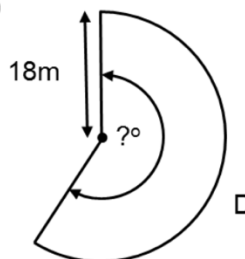
sector D is to have
an area of
639m²

what angle must
sector D have?

(5)  which sector (E or F)
has a bigger area?

you need to show your
calculations to justify
your answer

E

radius = 23cm
angle = 305°

F
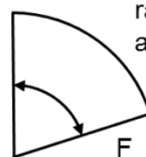
radius = 46cm
angle = 76°

9

**areas of sectors (ii)**

(1)

A

6cm

72°

B

12cm

?°
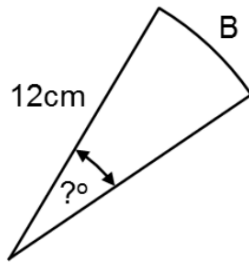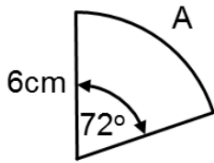
the two sectors A and B are to have exactly the same area

what angle must sector B have?

(2)

5cm

?°
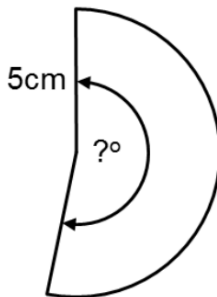
the sector (with a radius of 5cm) is to have an area of 40cm²

(a) what angle must the sector have?

what angle do you need for an area of 40cm² for a sector with radius (b) 4cm (c) 6cm ?

(give your answers correct to the nearest 0.1°)

(3)

?cm

32°
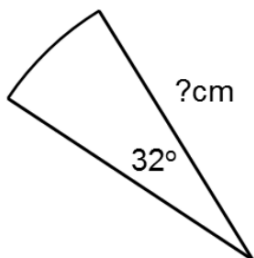
what radius must the sector have so that the area of the sector is 100cm² ?

(give your answer correct to two decimal places)

Shape and Space III (3) Further problem solving

**Do now:**

6cm

72°

12cm

?°
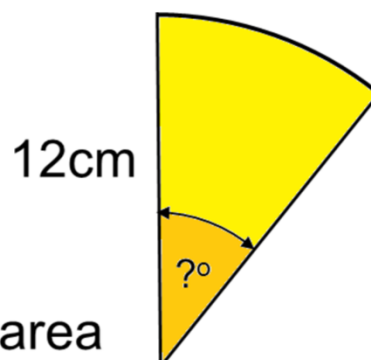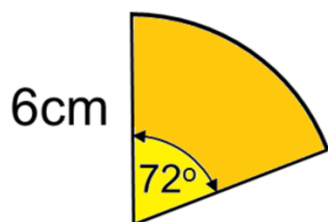
two sectors have the same area
what is the missing angle?

1      The diagram shows a shape made from a trapezium *ABCD* and a semicircle with diameter *DC*.

D    8 cm    C

5 cm

A      B
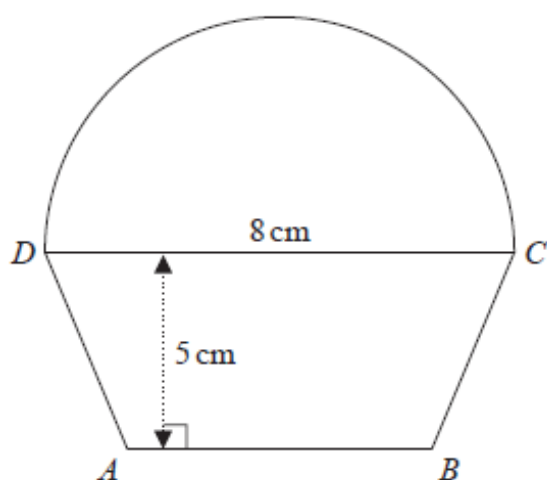
*DC* = 8 cm.
The shape has area 64 cm²
The height of the trapezium is 5 cm.

Work out the length of *AB*.
Give your answer correct to 1 decimal place.

....................................................... cm
**(Total for Question 1 is 5 marks)**

**2**     The diagram shows a cycle track.
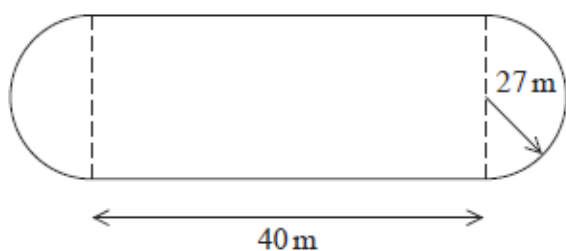


The track has two straight sides each of length 40 m.
Each end of the track is a semicircle of radius 27 m.

The diameter of each wheel of Ian's bike is 590 mm.
Ian is going to ride his bike around the track once.

Calculate how many complete revolutions each wheel of his bike will make.

......................................................

**(Total for Question 2 is 5 marks)**

_____

**3**     *OAB* is a sector of a circle with centre *O* and radius 7 cm.
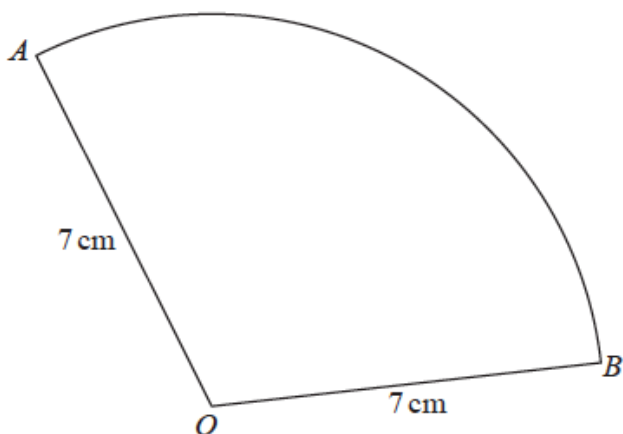


The area of the sector is 40 cm²
Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

.............................................. cm
**(Total for Question 3 is 4 marks)**

**4**
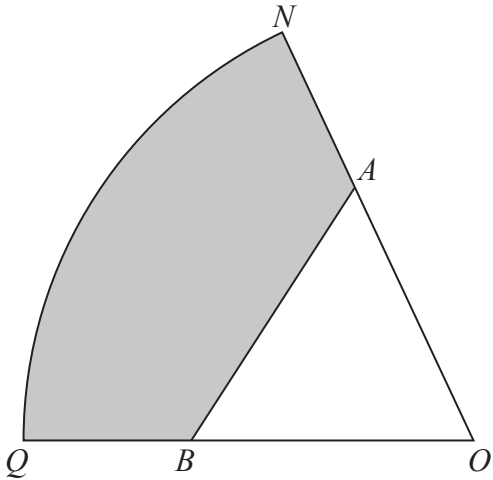


*ONQ* is a sector of a circle with centre *O* and radius 11 cm.
*A* is the point on *ON* and *B* is the point on *OQ* such that *AOB*
is an equilateral triangle of side 7 cm.
Calculate the area of the shaded region as a percentage of the area of the sector *ONQ*.
Give your answer correct to 1 decimal place.

......................................................%

**(Total for Question 4 is 5 marks)**

**5.** A square, with sides of length *x* cm, is inside a circle.
Each vertex of the square is on the circumference of the circle.
The area of the circle is 49 cm$^2$.
Work out the value of *x*.
Give your answer correct to 3 significant figures.

......................................................

**(Total for Question 5 is 4 marks)**

**6** The circumference of circle **B** is 90% of the circumference of circle **A**.

(*a*) Find the ratio of the area of circle **A** to the area of circle **B**.

......................................................

**(2)**

Square **E** has sides of length *e* cm.
Square **F** has sides of length *f* cm.
The area of square **E** is 44% greater than the area of square **F**.

(*b*) Work out the ratio *e* : *f*

......................................................

**(2)**

**7** The diagram shows a circle and an equilateral triangle.
One side of the equilateral triangle is a diameter of the circle.
The circle has a circumference of 44 cm.
Work out the area of the triangle.
Give your answer correct to 3 significant figures.
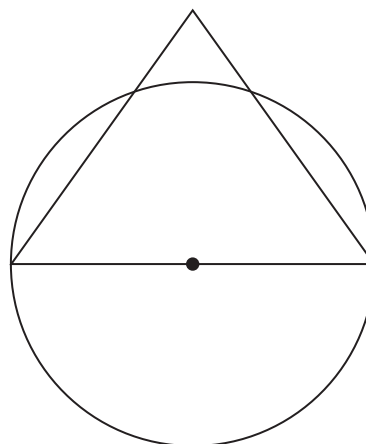


.......................................................cm$^2$
**(Total for Question 7 is 3 marks)**

**8**     The diagram shows 3 identical circles inside a rectangle.
Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram.
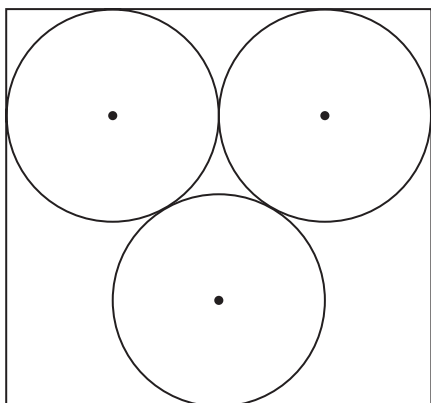


The radius of each circle is 24 mm.
Work out the area of the rectangle.
Give your answer correct to 3 significant figures.

.................................................. mm²
**(Total for Question 8 is 4 marks)**
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**9** The diagram shows a square *ABCD* with sides of length 20 cm.
It also shows a semicircle and an arc of a circle.



20 cm
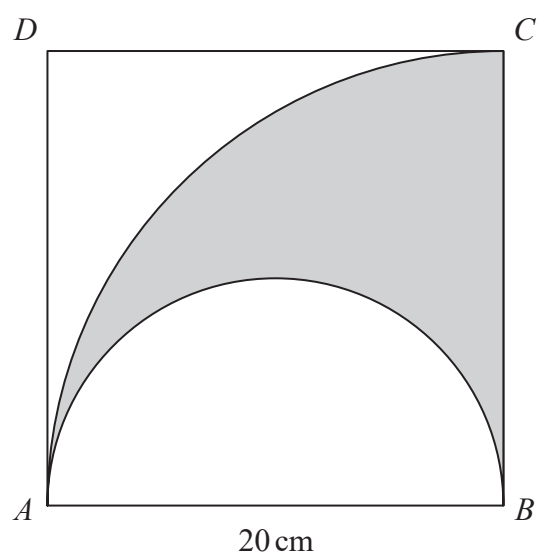
*AB* is the diameter of the semicircle.
*AC* is an arc of a circle with centre *B*.

Show that $\dfrac{\text{area of shaded region}}{\text{area of square}} = \dfrac{\pi}{8}$

**(Total for Question 9 is 4 marks)**

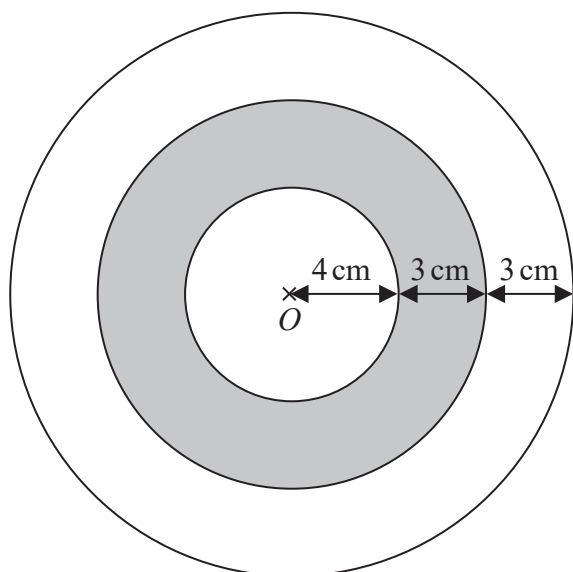**11** The diagram shows a logo made from three circles.
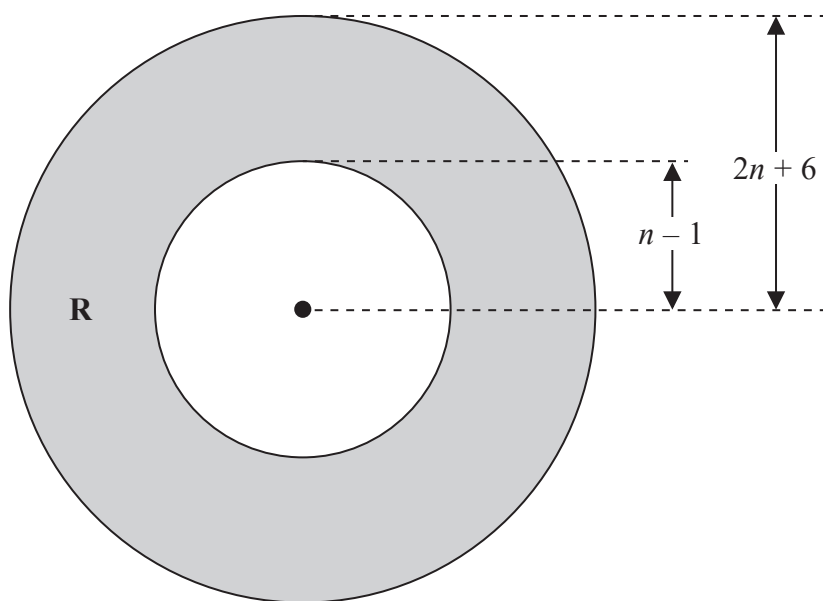


Each circle has centre *O*.

Daisy says that exactly $\dfrac{1}{3}$ of the logo is shaded.

Is Daisy correct?

You must show all your working.

**(Total for Question 11 is 4 marks)**

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**12** The region **R**, shown shaded in the diagram, is the region between two circles with the same centre.



The outer circle has radius $(2n + 6)$
The inner circle has radius $(n - 1)$
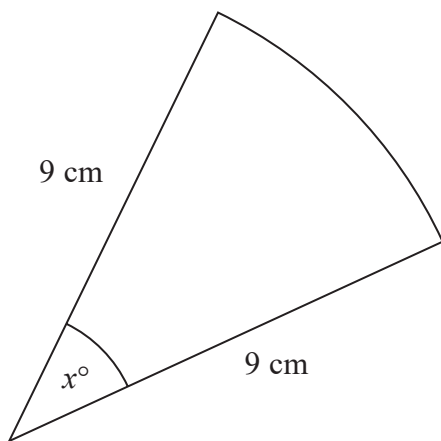All measurements are in centimetres.
The area of **R** is greater than the area of a circle of radius $(n + 13)$ cm.
$n$ is an integer.
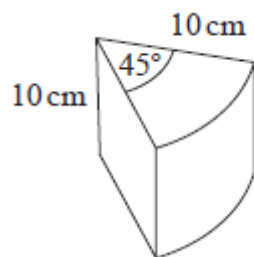Find the least possible value of $n$.
You must show all of your working.

.......................................................

**(Total for Question 12 is 5 marks)**

18

The sector has a perimeter of 25 cm.
Work out the value of *x*.
Give your answer correct to 1 decimal place.

......................................................
**(Total for Question 14 is 4 marks)**

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**16** Here are two solid prisms, prism **A** and prism **B**.
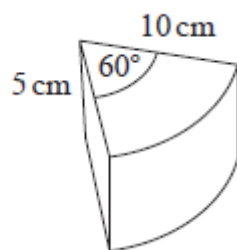


prism **A**                                   prism **B**

The cross section of prism **A** is a sector, with angle 45°, of a circle of radius 10 cm.
The prism has a depth of 10 cm and a mass of $40\pi$ grams.

The cross section of prism **B** is a sector, with angle 60°, of a circle of radius 10 cm.
The prism has a depth of 5 cm and a mass of $50\pi$ grams.

Express the difference in the densities of the two prisms as a percentage of the density of prism **A**.